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Dynamic analysis of plate heat exchangers with dispersion in both fluids

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Abstract—Based on a dispersion model, an extensive analysis is presented for counterflow plate heat exchangers, which takes the deviation from ideal plug flow into consideration to predict the response due to temperature transients. The analysis also incorporates solid longitudinal conduction and solid as well as fluid heat capacities, and simulates the exit temperatures of both fluids for any given arbitrary temperature transient at the inlet(s). It is found that the 'phase lag effect', which is a special characteristic of plate exchangers, plays a significant role in the dynamic regime. The examples are presented for step and sinusoidal response and the effect of U- and Z-type configurations has been discussed. The method is based on the Laplace transform and it utilizes numerical inversion of the Laplace transform. The results lead to the conclusion that the effect of flow maldistribution and 'phase lag' plays an important role, particularly when the decreasing flow velocity in the gasket ports is taken into consideration.

INTRODUCTION

Plate heat exchangers were first introduced in hygienic applications such as in the dairy and brewing industries to overcome the problem of cleaning and maintenance, which is important in such applications. However, in recent times they have attracted other users, such as in the chemical process industry and in heat recovery units. Apart from ease of maintenance, the other factor which has contributed to their fast growing popularity is their capability to generate higher turbulence at comparatively lower flow rates.

The literature dealing with the mathematical modelling of plate heat exchangers is vast. This includes the numerical models of Watson *et al.* [1] and Jackson and Troupe [2], where the Runge–Kutta integration method has been utilized, and the analytical model where a system of differential equations have been solved by the eigenvalue method, presented by Wolf [3], Buonopane *et al.* [4] and Zaleski [5]. Marano and Jechura [6] have refined the analytical model to make it more suitable for computer simulation. It is noticeable that all these analyses simulate the steady-state behaviour of plate heat exchangers. The transient analysis of plate heat exchangers is rather rare in the literature. McKnight and Worley [7] pioneered the study with the application of feedback control to high velocity flow. Zaleski and Tajszerski [8] simulated concurrent exchangers for transient response. Khan *et al.* [9] presented the transient analysis of a countercurrent plate heat exchanger subject to flow transient. They used a frequency response technique to obtain the related transfer function. From the experiment performed by them, they suggested a second-order transfer function with dead time. Another novel approach was taken by Lakshmann and Potter [10] by applying the 'cinematic model' developed by them. This method is also based on eigensystem analysis applied to the fluid path, divided into certain cells within the flow passage.

It is important to note that all the models mentioned above consider a plug flow of both fluids, ignoring the flow maldistribution or backmixing in them. This assumption is clearly a deviation from reality, since the effect of flow maldistribution in plate heat exchangers has been observed to be rather prominent by Amooie-Foumeny [11], Haseler *et al.* [12] and many others. The other effect which is typical of the plate heat exchangers is the 'phase lag effect' described by Roetzel *et al.* [13]. The flow maldistribution effect in shell-and-tube heat exchangers has recently been taken care of by introducing a dispersion term in the energy equation [14, 15]. The justification of considering such a term for plate heat exchangers can be observed from the experimental results presented recently [13].

The objective of the present analysis is to develop a transient model of single pass countercurrent (both U- and Z-type) plate heat exchangers by applying the dispersion model to take care of the flow maldistribution in channels. It is also important to observe the phase lag effect, which plays an important role in the transient regime. In this paper, firstly the governing differential equations have been derived with a general nomenclature chosen to describe the system. The equations are then transformed into matrix formulation using the method of Laplace transform. The solution to these equations has been obtained using the eigensystem analysis, taking proper boundary conditions into consideration. The results

NOMENCLATURE

- A heat transfer area per effective plate [m^2]
- $\mathbf{A}$ coefficient matrix for system of differential equations
- A_c free flow area in a channel [m^2]
- $A_{k,m}$ the element of kth row and mth column of matrix $\mathbf{A}$
- A_w cross-sectional area of the plate available for longitudinal conduction [m^2]
- $\mathbf{B}$ diagonal matrix, equation (41)
- C heat capacity of fluid(s) [J K^{-1}]
- C_w heat capacity of plate material [J K^{-1}]
- D axial dispersion coefficient [W m^{-1} K^{-1}]
- $\mathbf{D}$ matrix resulting from boundary conditions, equation (48)
- d_j elements of matrix $\mathbf{D}$
- $f(Z)$ inlet temperature function
- $F(S)$ Laplace transform of $f(Z)$
- h heat transfer coefficient [W m^{-2} K^{-1}]
- i square root of -1
- l_i path traversed by fluid particle before entering ith channel [m]
- L fluid flow length in channels [m]
- m_j $j-2[j/2]$, where j is an integer
- n number of channels on one side
- N number of channels
- NTU_1 number of transfer units, $[(1/U_1)+(n_1/n_2)(1/R_2U_2)]^{-1}$
- Pe axial dispersive Péclet number, $\dot{w}L/A_cD$
- R_2 capacity rate ratio in the channels, $\dot{w}_2/\dot{w}_1$
- R_{g2} capacity rate ratio of the combined flow, $\dot{w}_{g2}/\dot{w}_{g1}$
- $R_{g\tau}$ characteristic time ratio of the combined fluid, τ_{rg2}/τ_{rg1}
- R_N ratio U_2/U_1
- R_{pe} ratio of Péclet numbers in channels, Pe_2/Pe_1
- R_w wall heat capacity ratio, C_w/C_1
- R_τ characteristic time ratio in channels, τ_{r2}/τ_{r1}
- S transformed time variable in Laplace domain
- $\mathbf{S}$ matrix with inlet fluid function, equation (48)
- t dimensionless temperature, $(\theta-\theta_{g1,\mathrm{in}})/(\theta_{g2,\mathrm{in}}-\theta_{g1,\mathrm{in}})$
- T temperature obtained by Laplace transformation of temperature t
- $\mathbf{T}$ temperature matrix, equation (39)
- $_{\mathrm{in}}(\Delta t)_{\mathrm{out}}$ the temperature change of a fluid from inlet to outlet [K]
- u a unit step function
- $\mathbf{U}$ matrix of eigenvectors of the matrix $\mathbf{A}$
- $u_{i,j}$ elements of matrix $\mathbf{U}$
- $U_{1(2)}$ $(hA/\dot{w})_{1(2)}$
- $V_{i'}$ the velocity of fluid in the gasket port after ith channel [m s^{-1}]
- V_g velocity of the combined fluid [m s^{-1}]
- $\dot{V}$ volume flow rate [m^3 s^{-1}]
- $\dot{w}$ thermal capacity rate of fluid in channels [W K^{-1}]
- $\dot{w}_g$ thermal capacity rate of combined fluid [W K^{-1}]
- X space coordinate [m]
- x dimensionless space coordinate, X/L
- Z dimensionless time, τ/τ_{r1}.

Greek symbols

- β_j jth eigenvalue of matrix $\mathbf{A}$
- γ_w wall conduction parameter, $A_w\lambda_w/\dot{w}_1L$
- θ temperature [K]
- τ time [s]
- $\Delta\tau$ time of travel in the port (between channels) [s]
- λ_w thermal conductivity of plate in longitudinal direction [W m^{-1} K^{-1}]
- τ_r characteristic time, $C/\dot{w}$
- ϕ dimensionless phase lag (cumulative value)
- $\Delta\phi$ dimensionless phase lag (discrete value).

Subscripts

- exit at exit
- g combined flow before splitting in channels or after recombination at exit
- i ith channel
- in at inlet
- w plate
- wi ith plate
- 0 initial
- 1 the fluid in odd channels
- 2 the fluid in even channels.

of the simulation demonstrate the effect of dispersion, phase lag effect and end effect, which are characteristics of plate heat exchangers.

It is important to mention that, in the present model, the thermal capacity of the plates and the longitudinal heat conduction in them have been taken into consideration to make analysis more realistic. The model can be used as a tool to impart proper

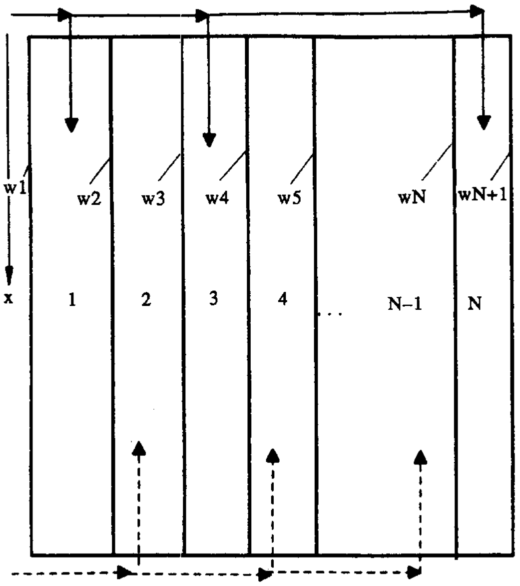

Fig. 1. Channel and flow configuration, w1, w2, ... wN+1 indicate the N+1 plates and 1, 2, ... N indicate N channels (odd number of channels assumed).

control to the plate heat exchanger, which is developing as an alternative to many other heat exchange equipments.

MATHEMATICAL FORMULATION

To model a plate heat exchanger mathematically, it is necessary to resort to certain assumptions. These assumptions are in line with the assumptions made for the transient analysis of shell-and-tube heat exchangers. The additional assumptions arise from the dispersion effect, which takes care of the deviation from the plug flow model and from the phase lag effect which is described later. The assumptions can be listed as:

(1) all flow and thermal properties are constant;

(2) the flow velocity and the mean heat transfer coefficient are identical for the channels carrying similar fluids but may be different for the two fluids;

(3) the thermal resistance is zero across the width and thickness of the plates, but it is finite along the plate length;

(4) heat transfer takes place only across the plates and not through the sealing edges or gaskets;

(5) the heat exchanger is thermally insulated from the atmosphere;

(6) the flow maldistribution in the flow passages can be described by introducing a dispersion term into the energy equation;

(7) the exchanger is started from a uniform temperature.

With these assumptions the counterflow single pass plate heat exchanger can be represented schematically, as shown in Fig. 1. The coordinate system is chosen in the direction of flow through the first channel. The channels are named from 1 to N and the plates 1 to $N+1$, as shown in the figure, where an odd number of channels is assumed, for an even number of channels the $(N-1)$th and Nth channel will carry fluid 1 and fluid 2, respectively. With this nomenclature, it is important to note that each plate exchanges heat with fluids flowing on both of its sides excepting the 1st and $(N+1)$th plate, which are in contact with one fluid only. The other sides of these two plates are open to the atmosphere and assumed to be insulated. Taking energy balance over differential elements in the channels and plates and considering dispersion in both the fluids, the following governing equations can be derived:

$$\frac{C_1}{L}\frac{\partial\theta_i}{\partial\tau} = A_c D_1 \frac{\partial^2\theta_i}{\partial X^2} - (-1)^{i-1}\dot{w}_1\frac{\partial\theta_i}{\partial X} + \frac{(hA)_1}{2L}\times(\theta_{wi}+\theta_{wi+1}-2\theta_i) \quad \left(i = 1,3,5\ldots 2\left[\frac{N+1}{2}\right]-1\right) \tag{1}$$

$$\frac{C_2}{L}\frac{\partial\theta_i}{\partial\tau} = A_c D_2 \frac{\partial^2\theta_i}{\partial X^2} - (-1)^{i-1}\dot{w}_2\frac{\partial\theta_i}{\partial X} + \frac{(hA)_2}{2L}\times(\theta_{wi}+\theta_{wi+1}-2\theta_i) \quad \left(i = 2,4,6\ldots 2\left[\frac{N}{2}\right]\right) \tag{2}$$

$$\frac{C_w}{L}\frac{\partial\theta_{wi}}{\partial\tau} = \lambda_w A_w \frac{\partial^2\theta_{wi}}{\partial X^2} + \frac{(hA)_1}{2L}(\theta_{i-1}-\theta_{wi}) + \frac{(hA)_2}{2L}(\theta_i-\theta_{wi}) \quad \left(i = 2,4,6\ldots 2\left[\frac{N+1}{2}\right]\right) \tag{3}$$

$$\frac{C_w}{L}\frac{\partial\theta_{wi}}{\partial\tau} = \lambda_w A_w \frac{\partial^2\theta_{wi}}{\partial X^2} + \frac{(hA)_2}{2L}(\theta_{i-1}-\theta_{wi}) + \frac{(hA)_1}{2L}(\theta_i-\theta_{wi}) \quad \left(i = 3,5,7\ldots 2\left[\frac{N}{2}\right]-1\right) \tag{4}$$

$$\frac{C_w}{L}\frac{\partial\theta_{w1}}{\partial\tau} = \lambda_w A_w \frac{\partial^2\theta_{w1}}{\partial X^2} + \frac{(hA)_1}{2L}(\theta_1-\theta_{w1}) \tag{5}$$

$$\frac{C_w}{L}\frac{\partial\theta_{wN+1}}{\partial\tau} = \lambda_w A_w \frac{\partial^2\theta_{wN+1}}{\partial X^2} + \frac{(hA)_*}{2L}(\theta_N-\theta_{wN+1}) \tag{6}$$

where $(hA)_* = (hA)_1$ for odd N; $= (hA)_2$ for even N.

The large number of variables involved in these equations can be reduced to a few dimensionless groupings which are conventionally used to characterize heat exchangers. The most important amongst these groups are the number of transfer units NTU, heat capacity rate ratio R_2 and Péclet number Pe, characterizing heat transfer, thermal balance and dispersion in fluids, respectively. The temperature, time and spatial coordinates can also be reduced to dimensionless form. The total set of dimensionless variables chosen for the present analysis can be listed as

$$\tau_{r1} = \frac{C_1}{\dot{W}_1} \quad \tau_{r2} = \frac{C_2}{\dot{W}_2}$$

$$U_1 = \frac{(hA)_1}{\dot{W}_1} \quad U_2 = \frac{(hA)_2}{\dot{W}_2}$$

$$NTU_1 = \left[\frac{1}{U_1} + \frac{n_1}{n_2}\frac{1}{R_2 U_2}\right]^{-1}$$

$$R_2 = \frac{\dot{W}_2}{\dot{W}_1} \quad R_w = \frac{C_w}{C_1} \quad R_\tau = \frac{\tau_{r2}}{\tau_{r1}} \quad R_N = \frac{U_2}{U_1}$$

$$Pe_1 = \frac{\dot{W}_1 L}{A_c D_1} \quad Pe_2 = \frac{\dot{W}_2 L}{A_c D_2}$$

$$\gamma_w = \frac{\lambda_w A_w}{\dot{W}_1 L}$$

$$x = X/L \quad Z = \tau/\tau_{r1} \quad t = \frac{\theta - \theta_{g1,\mathrm{in}}}{\theta_{g2,\mathrm{in}} - \theta_{g1,\mathrm{in}}}.$$

With these dimensionless parameters, equations (1)–(6) can be recast in the following non-dimensional form:

$$R_\tau^{m_{i+1}} \frac{\partial t_i}{\partial Z} = \frac{1}{Pe_1 (R_{pe})^{m_{i+1}}} \frac{\partial^2 t_i}{\partial x^2} - (-1)^{i-1} \frac{\partial t_i}{\partial x} + \frac{U_1 R_N^{m_{i+1}}}{2}(t_{wi} + t_{wi+1} - 2t_i) \quad (i = 1,2,3 \ldots N) \tag{7}$$

$$R_w \frac{\partial t_{wi}}{\partial Z} = \gamma_w \frac{\partial^2 t_{wi}}{\partial x^2} + \frac{U_1}{2}(R_N R_2)^{m_i}(t_{i-1} - t_{wi}) + \frac{U_1}{2}(R_N R_2)^{m_{i+1}}(t_i - t_{wi}) \quad (i = 2,3,4 \ldots N) \tag{8}$$

$$R_w \frac{\partial t_{w1}}{\partial Z} = \gamma_w \frac{\partial^2 t_{w1}}{\partial x^2} + \frac{U_1}{2}(t_1 - t_{w1}) \tag{9}$$

$$R_w \frac{\partial t_{wN+1}}{\partial Z} = \gamma_w \frac{\partial^2 t_{wN+1}}{\partial x^2} + \frac{U_1}{2}(R_N R_2)^{m_{N+1}}(t_N - t_{wN+1}) \tag{10}$$

where $m_j = j - 2[j/2]$.

It is important to note that the various ratios used in these equations conform to their values in channels and not the total combined values of the flowing fluids. They are related to the values of ratios of the combined total stream as

$$R_2 = R_{g2}\left(\frac{n_1}{n_2}\right) \tag{11}$$

$$R_\tau = R_{g\tau}\left(\frac{n_2}{n_1}\right). \tag{12}$$

The Péclet number used for the analysis corresponds to its value within the channels, because in the present model the dispersion is considered to take place only within the channels and not in the conduits carrying the fluids to the channels.

THE PHASE LAG EFFECT AND THE BOUNDARY CONDITIONS

The special feature, which makes the plate heat exchangers different from shell-and-tube heat exchanger in respect of its entry condition in the transient regime, is the 'phase lag effect'. This means that the fluid enters channel 1, 2, 3 ... at an increasing phase lag from the time at which the combined flow enters the heat exchanger at point 1 (Fig. 2). In a U-type plate heat exchanger this effect gets even more increased, because the fluids from channels 1, 2, 3 ... also encounter increasing amount of time delay before they mix up and the combined stream reaches the exit point 2. In a Z-type plate heat exchanger the situation is different. Here, unlike a U-type exchanger, each fluid stream travels an equal length of path within the heat exchanger. This means that the effect of phase lag at the entry of the channel is reduced by the decreasing phase lag of the streams from channels 1, 2, 3 .... This effect has been assumed to be incorporated in the value of Péclet number in the previous study [13], which makes it particular to a special type of plate heat exchanger and obviously not applicable for the present differential model of the equipment. Hence a more accurate model specifying the entry condition to each channel has been utilized here.

The phase lag before entry to the channels can be calculated keeping in mind that the flow rate in each channel carrying similar fluid has been assumed to be equal. Since the flow areas of the conduits carrying fluids to the channels are same, so the velocity of flow decreases in the conduit after every channel stream leaves the conduit (due to decrease in volume flow rate). This is shown in Fig. 3, where the velocities $V_{1'}$, $V_{2'}$, $V_{3'}$, ... are the velocities in the conduit after channels 1, 2, 3 ..., respectively. From the continuity condition the ratios of these velocities with the entrance velocity may be derived as

$$\frac{V_{(2i-1)'}}{V_{g1}} = 1 - i\frac{\dot{V}_1}{\dot{V}_{g1}} = 1 - \frac{i}{n_1} \quad (\text{for } i = 1,2,3 \ldots x_1) \tag{13}$$

$$\frac{V_{2i'}}{V_{g2}} = 1 - i\frac{\dot{V}_2}{\dot{V}_{g2}} = 1 - \frac{i}{n_2} \quad (\text{for } i = 1,2,3 \ldots x_2). \tag{14}$$

The time required for the fluid to travel the distance between the channels can be calculated as (distances shown in Fig. 2)

$$\Delta\tau_1 = l_1/V_{g1} \tag{15}$$

$$\Delta\tau_2 = l_2/V_{g2} \tag{16}$$

$$\Delta\tau_{2i+1} = (l_{2i+1} - l_{2i-1})/V_{(2i-1)'} \quad (i = 1,2,3 \ldots (n_1 - 1)) \tag{17}$$

$$\Delta\tau_{2i+2} = (l_{2i+2} - l_{2i})/V_{2i'} \quad (i = 1,2,3 \ldots (n_2 - 1)). \tag{18}$$

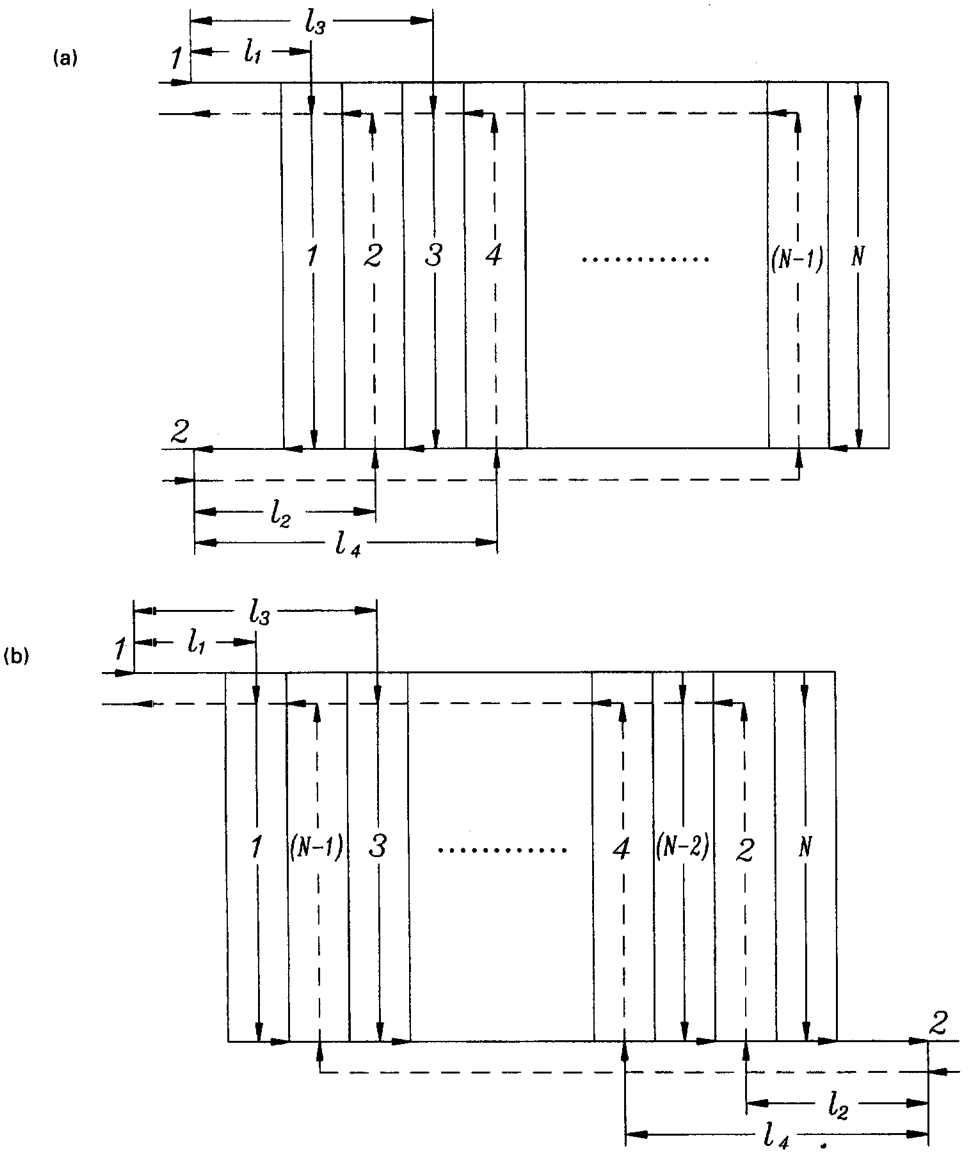

Fig. 2. Flow lengths before entering into and after exit from channels in plate heat exchangers. (a) U-type, (b) Z-type.

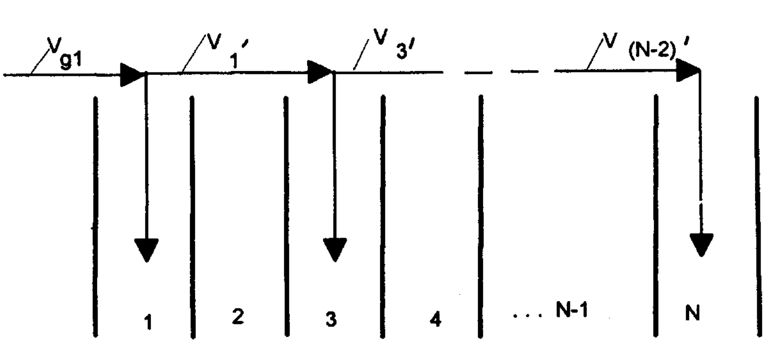

Fig. 3. Varying velocity in the port after flow departure in channels of side 1 (odd number of channels assumed).

Hence the dimensionless phase lag between the entrance of consecutive channels may be expressed as

$$\Delta\phi_i = \Delta\tau_i/\tau_{r1}.$$

The total phase lag at the entry of each channel is the cumulated sum of the phase lags given by

$$\phi_{2i-1} = \sum_{j=1}^{2i-1} \Delta\phi_{2j-1} \quad (i = 1,2\ldots n_1)$$

$$\phi_{2i} = \sum_{j=1}^{2i} \Delta\phi_{2j} \quad (i = 1,2\ldots n_2). \tag{19}$$

The phase lag encountered at the exit of the channels to arrive at the exit point can be computed in a similar way. Under the condition of dimensional symmetry in construction, as shown in Fig. 2, the relationship for this phase lag at exit reduces to

$$\phi_{i,\text{exit}} = \phi_i \tag{20}$$

for a U-type plate exchanger, and

$$\phi_{i,\text{exit}} = \phi_{n-i-1} \tag{21}$$

for a Z-type plate exchanger.

With this definition of phase lag, the boundary conditions for equations (7)–(10) may be written in accordance with Danckwerts [16] as:

at $x = 0$:

$$t_i - \frac{1}{Pe_1}\frac{\partial t_i}{\partial x} = f_1(Z-\phi_i)u(Z-\phi_i)$$

$$\left(i = 1,3,5\ldots 2\left[\frac{N+1}{2}\right]-1\right) \tag{22}$$

$$\frac{\partial t_i}{\partial x} = 0 \quad \left(i = 2,4,6\ldots 2\left[\frac{N}{2}\right]\right) \tag{23}$$

$$\frac{\partial t_{wi}}{\partial x} = 0 \quad (i = 1,2,3\ldots N+1) \tag{24}$$

at $x = 1$:

$$t_i + \frac{1}{Pe_1 R_{pe}}\frac{\partial t_i}{\partial x} = f_2(Z-\phi_i)u(Z-\phi_i)$$

$$\left(i = 2,4,6\ldots 2\left[\frac{N}{2}\right]\right) \tag{25}$$

$$\frac{\partial t_i}{\partial x} = 0 \quad \left(i = 1,3,5\ldots 2\left[\frac{N+1}{2}\right]-1\right) \tag{26}$$

$$\frac{\partial t_{wi}}{\partial x} = 0 \quad (i = 1,2,3\ldots N+1). \tag{27}$$

SOLUTION FOR TEMPERATURE DISTRIBUTION

The $2N+1$ partial differential equations expressed by equations (7)–(10), along with the boundary conditions (22)–(27), describe the mathematical model presented in the current analysis. The initial condition for both the fluids and the walls may be taken to be uniform, since only starting from uniform temperature (cold state) is considered. Thus

$$t_{i,0} = t_{wi,0} = 0.$$

This system of differential equations can be solved by taking the Laplace transform with respect to the reduced time variable Z. With this operation, the system of equations (7)–(10) is transformed into

$$\frac{d^2 T_i}{dx^2} = Pe_1 R_{pe}^{m_{i+1}} (R_\tau^{m_{i+1}} S + U_1 R_N^{m_{i+1}})\, T_i$$

$$+(-1)^{i-1} Pe_1 R_{pe}^{m_{i+1}} \frac{dT_i}{dx} - \frac{U_1}{2}\cdot Pe_1\, (R_{pe} R_N)^{m_{i+1}}\, T_{wi}$$

$$-\frac{U_1}{2} Pe_1\, (R_{pe} R_N)^{m_{i+1}}\, T_{wi+1} \tag{28}$$

$$\frac{d^2 T_{wi}}{dx^2} = \gamma_w^{-1}\left\{R_w S + \frac{U_1}{2}(R_N R_2)^{m_i} + \frac{U_1}{2}(R_N R_2)^{m_{i+1}}\right\} T_{wi}$$

$$-\gamma_w^{-1}\frac{U_1}{2}(R_N R_2)^{m_i}\, T_{i-1} - \gamma_w^{-1}\frac{U_1}{2}(R_N R_2)^{m_{i+1}}\, T_i \tag{29}$$

$$\frac{d^2 T_{w1}}{dx^2} = \gamma_w^{-1}\left(R_w S + \frac{U_1}{2}\right) T_{wi} - \gamma_w^{-1}\frac{U_1}{2} T_1 \tag{30}$$

$$\frac{d^2 T_{wN+1}}{dx^2} = \gamma_w^{-1}\left\{R_w S + \frac{U_1}{2}(R_N R_2)^{m_{N+1}}\right\}$$

$$\times T_{wN+1} - \gamma_w^{-1}\frac{U_1}{2}(R_N R_2)^{m_{N+1}} T_N. \tag{31}$$

Similarly the boundary conditions, equations (22)–(27), can also be transformed into:

at $x = 0$:

$$T_i - \frac{1}{Pe_1}\frac{dT_i}{dx} = F_1(S)\, e^{-\phi_i s}$$

$$\left(i = 1,3,5\ldots 2\left[\frac{N+1}{2}\right]-1\right) \tag{32}$$

$$\frac{dT_i}{dx} = 0 \quad \left(i = 2,4,6\ldots 2\left[\frac{N}{2}\right]\right) \tag{33}$$

$$\frac{dT_{wi}}{dx} = 0 \quad (i = 1,2,3\ldots N+1) \tag{34}$$

at $x = 1$:

$$T_i + \frac{1}{Pe_1 R_{pe}}\frac{dT_i}{dx} = F_2(S)\, e^{-\phi_i s}$$

$$\left(i = 2,4,6\ldots 2\left[\frac{N}{2}\right]\right) \tag{35}$$

$$\frac{dT_i}{dx} = 0 \quad \left(i = 1,3,5\ldots 2\left[\frac{N+1}{2}\right]-1\right) \tag{36}$$

$$\frac{dT_{wi}}{dx} = 0 \quad (i = 1,2,3\ldots N+1). \tag{37}$$

The system of transformed equations (28)–(31) can be expressed by matrix notation as

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}x} = \mathbf{AT} \tag{38}$$

where the vector $\mathbf{T}$ is given by

$$\mathbf{T} = \left(T_1, T_2, \ldots, T_N, \frac{\mathrm{d}T_1}{\mathrm{d}x}, \frac{\mathrm{d}T_2}{\mathrm{d}x}, \ldots, \frac{\mathrm{d}T_N}{\mathrm{d}x}, \right.$$

$$\left. T_{\mathrm{w}1}, T_{\mathrm{w}2}, \ldots, T_{\mathrm{w}N+1}, \frac{\mathrm{d}T_{\mathrm{w}1}}{\mathrm{d}x}, \frac{\mathrm{d}T_{\mathrm{w}2}}{\mathrm{d}x}, \ldots, \frac{\mathrm{d}t_{\mathrm{w}N+1}}{\mathrm{d}x} \right)^{\mathrm{T}} \tag{39}$$

and the coefficient matrix $\mathbf{A}$ can be written as

$$A_{i,N+i} = 1 \quad A_{N+i,i} = Pe_1 (R_\tau^{m_{i+1}} S + U_1 R_N^{m_{i+1}}) R_{pe}^{m_{i+1}}$$

$$A_{N+i,N+i} = (-1)^{i-1} Pe_1 (R_{pe})^{m_{i+1}}$$

$$A_{N+i,2N+i} = -Pe_1 (R_{pe} R_N)^{m_{i+1}} \frac{U_1}{2}$$

$$A_{N+i,2N+i+1} = -Pe_1 (R_{pe} R_N)^{m_{i+1}} \frac{U_1}{2}$$

$$(i = 1, 2, 3 \ldots N)$$

$$A_{2N+i,3N+1+i} = 1 \quad (i = 1, 2, 3 \ldots N+1)$$

$$A_{3N+2,1} = -\gamma_{\mathrm{w}}^{-1} \frac{U_1}{2}$$

$$A_{3N+2,2N+1} = \left(R_{\mathrm{w}} S + \frac{U_1}{2} \right) \gamma_{\mathrm{w}}^{-1}$$

$$A_{4N+2,N} = -\gamma_{\mathrm{w}}^{-1} (R_N R_2)^{m_{N+1}} \frac{U_1}{2}$$

$$A_{4N+2,3N+1} = \gamma_{\mathrm{w}}^{-1} \left\{ R_{\mathrm{w}} S + \frac{U_1}{2} (R_N R_2)^{m_{N+1}} \right\}$$

$$A_{3N+1+i,i} = -\gamma_{\mathrm{w}}^{-1} \frac{U_1}{2} (R_N R_2)^{m_{i+1}}$$

$$A_{3N+1+i,i-1} = -\gamma_{\mathrm{w}}^{-1} \frac{U_1}{2} (R_N R_2)^{m_i}$$

$$A_{3N+1+i,2N+1} = \gamma_{\mathrm{w}}^{-1} \left\{ R_{\mathrm{w}} S + \frac{U_1}{2} (R_N R_2)^{m_i} + \frac{U_1}{2} (R_N R_2)^{m_{i+1}} \right\}$$

$$(i = 2, 3, 4 \ldots N).$$

All elements of matrix $\mathbf{A}$ other than those described above are zero in magnitude.

The solution to equation (38) can be obtained by deriving the eigenvalues β_j and eigenvectors $[u_j]$ of the coefficient matrix $\mathbf{A}$. This is a simple boundary value problem with distance coordinate as the only variable. The solution is

$$\mathbf{T} = \mathbf{UB}(x)\mathbf{D} \tag{40}$$

where $\mathbf{B}(x)$ is a diagonal matrix:

$$\mathbf{B}(x) = \mathrm{diag}\,\{\mathrm{e}^{\beta_1 x}, \mathrm{e}^{\beta_2 x}, \ldots, \mathrm{e}^{\beta_{4N+2} x}\}. \tag{41}$$

The matrix $\mathbf{U}$ is one whose columns are corresponding eigenvectors of $\mathbf{A}$, and $\mathbf{D}$ is a coefficient vector which depends on the boundary conditions given by equations (32)–(37). The fluid and wall temperature distribution can thus be expressed as

$$T_i = \sum_{j=1}^{4N+2} d_j u_{ij} \mathrm{e}^{\beta_j x} \tag{42}$$

$$T_{\mathrm{w}i} = \sum_{j=1}^{4N+2} d_j u_{N+i,j} \mathrm{e}^{\beta_j x}. \tag{43}$$

The derivatives of these temperatures can be expressed as

$$\frac{\mathrm{d}T_i}{\mathrm{d}x} = \sum_{j=1}^{4N+2} d_j u_{N+i,j} \mathrm{e}^{\beta_j x} \tag{44}$$

$$\frac{\mathrm{d}T_{\mathrm{w}i}}{\mathrm{d}x} = \sum_{j=1}^{4N+2} d_j u_{3N+1+i,j} \mathrm{e}^{\beta_j x}. \tag{45}$$

The coefficient matrix $\mathbf{D}$ can be determined by applying equations (42)–(45) to the boundary conditions (32)–(37) to obtain the matrix equation

$$\mathbf{WD} = \mathbf{S}. \tag{46}$$

Obviously the right-hand vector $\mathbf{S}$ contains the inlet temperature function along with its phase lag and can be written as

$$\mathbf{S} = [F_1(S)\,\mathrm{e}^{-\phi_1 s}, F_2(S)\,\mathrm{e}^{-\phi_2 s}, F_1(S)\,\mathrm{e}^{-\phi_3 s},$$

$$F_2(S)\,\mathrm{e}^{-\phi_4 s}, \ldots, F_K(S)\,\mathrm{e}^{-\phi_N s}, 0, 0, 0 \ldots 0] \tag{47}$$

where $K = 1$ for N odd; $= 2$ for N even.

Hence $\mathbf{D}$ can be obtained from

$$\mathbf{D} = \mathbf{W}^{-1}\mathbf{S}. \tag{48}$$

However, the solution expressed by equations (42) and (43) exists only for distinct eigenvalues β_j. For multiple eigenvalues the method of adding small parameters as expressed in ref. [14] can be adopted.

TEMPERATURE RESPONSE

The solution obtained in the preceding section expresses the temperature response in the frequency domain. To obtain the response in the time domain, an inverse Laplace transform is performed. It is implicit that the only way of inverting the complex solution $\mathbf{T}$ is by the numerical method. In this paper the Fourier series approximation method for numerical inversion of Laplace transform [17] has been used, because it is applicable to both step and oscillatory responses. For any function $g(Z)$ with a Laplace transform $G(S)$ it can be expressed as

$$g(Z) = \frac{\exp(aZ)}{Z} \left[\frac{1}{2} G(a) + \mathrm{Re} \sum_{k=i}^{\infty} G\left(a + \frac{ik\pi}{Z} \right) (-1)^k \right]. \tag{49}$$

The constant a is chosen in the domain $4 < aZ < 5$

to minimize the truncation error. For a plate exchanger, since the response from each channel is to be calculated at the time when it reaches the combined exit, it becomes very time consuming to calculate the response from equation (49). It can be further simplified by using Fast Fourier Transform. Substituting $Z = 2nZ/M$, equation (49) yields

$$g(Z_n) = \frac{\exp(aZ_n)}{Z}\left[\mathrm{Re}\sum_{k=0}^{M-1}{}' G\left(a+\frac{ik\pi}{Z}\right)\times\exp\left(i\frac{2\pi nk}{M}\right)-\frac{1}{2}g(a)\right]. \quad (50)$$

The term indicated by summation is obtained by

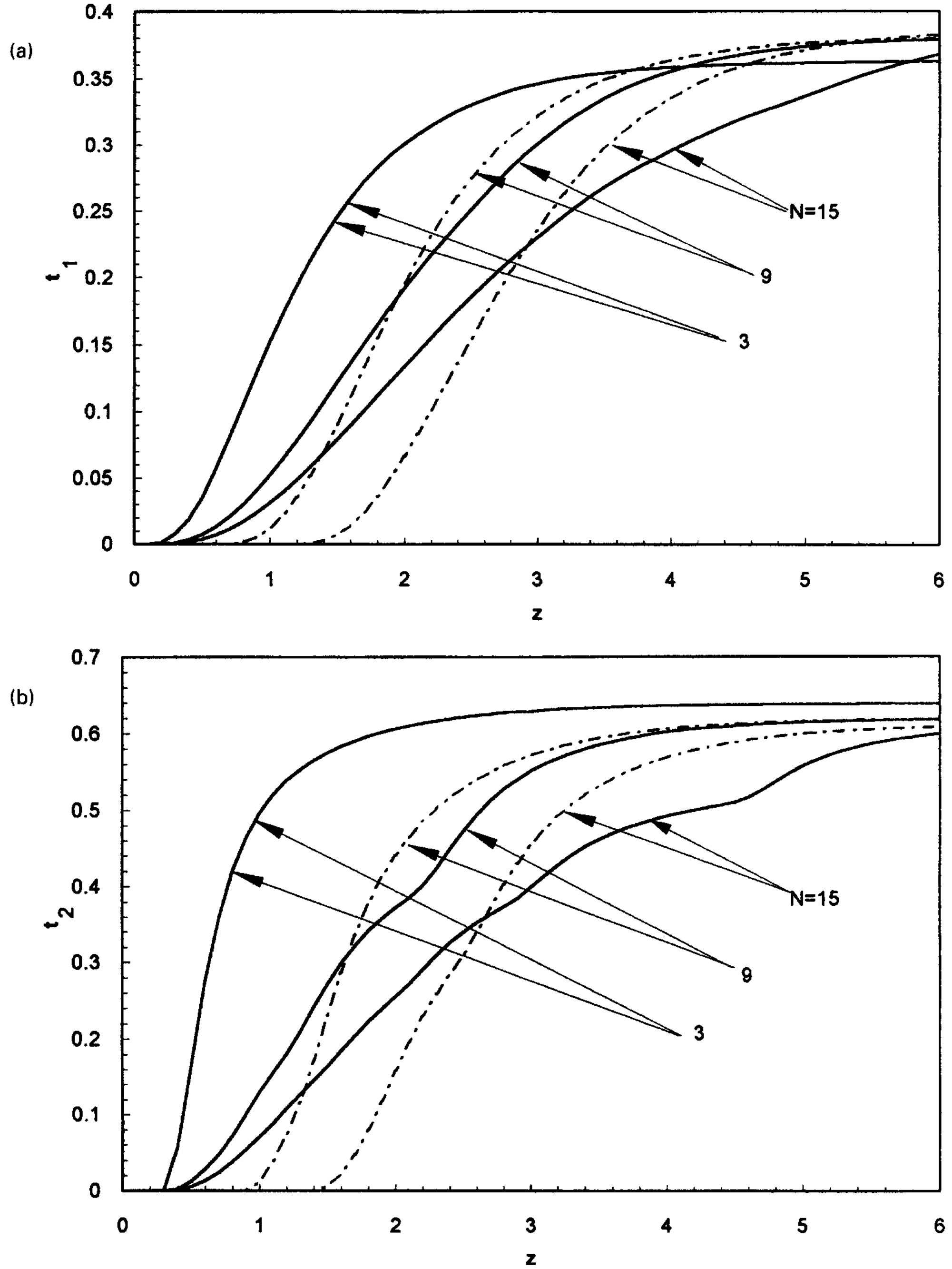

Fig. 4. Effect of number of channels on the exit response due to a step change of the inlet temperature of side 2. $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(z) = 0.0$, $f_2(z) = 1.0$, $NTU_1 = 1.0$, $\gamma_w = 0.1$ and $Pe = 5.0$. (a) Fluid 1, (b) fluid 2; (———) U-type, (–·–·–·) Z-type.

Fast Fourier Transform at every point Z_n in the assigned domain. With this algorithm, the step response results the steady-state response for $Z \to \infty$. This, in fact, can be used as a test for correctness of the simulation, since

$$_{\text{in}}(\Delta t_1)_{\text{out}} = R_{g2_{\text{in}}}(\Delta t_2)_{\text{out}} \quad \text{at steady state.}$$

RESULTS AND DISCUSSION

By applying the above-mentioned method, the response for all sorts of temperature transients can be calculated due to change in either of the fluid inlet temperatures (or both). Some examples are presented here. The examples are chosen for realistic values of heat exchanger parameters such as NTU, R_{g2} and geometrical configuration. The entry temperature of fluid 2 is assumed to change with time. The main two types of change, namely, step and oscillation, which are the most common types of disturbances, have been chosen as examples. The plate spacing has been considered to be 4% of the effective length of the plates, and the circular inlet and outlet ports where flow bifurcation and mixing occur have been chosen to give equal Péclet number in all the channels. However, the method applies equally well to cases with different Peclét number in hot and cold channels.

With the values of $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0, NTU_1 = 1.0, \gamma_w = 0.1$ and $Pe = 5.0$, the step responses have been calculated for a varying number of channels so as to get the effect of 'phase lag', which increases with number of channels. These results are depicted in Fig. 4. It is interesting to note that, in all the cases, the outlet temperatures of hot and cold fluids differ for U- and Z-type configuration, and the difference between the two types increases with the number of channels. This is expected because, with increasing number of channels, the 'phase lag effect' increases, which makes the U-type plate exchanger quite different from the Z-type. It is also important to note that, in U-type plate exchangers, the time lag in temperature response is less than that in the Z-type, which is also explicable from the fact that in Z-type configuration the fluid has to travel across all the channels to give a temperature increase at the outlet, while in U-type the temperature increase is sensed earlier at the outlet due to a quicker response from nearer channels. One interesting point to note is the outlet temperature of the hot fluid. Here it is observed that, in U-type configuration with higher numbers of channels, the response does not rise smoothly to the steady-state value, but shows some oscillations. This feature is found to be absent in Z-type configurations. With an extensive survey of results, it has been noted that any fluid which undergoes added up phase lag in entry and exit undergoes such swings. To clarify this, examples are presented for the case where a step change has occurred in both fluid entry temperatures (Fig. 5). Under such conditions the exit temperatures of both fluids are found to swing before attaining a steady-state value. In all the cases this feature has been found to be absent in Z-type and in the fluid which does not experience phase lag at the entry. To draw a conclusion about the origin of such swings, further sets of examples have been presented in Fig. 6, where the hot side fluid exit temperature is

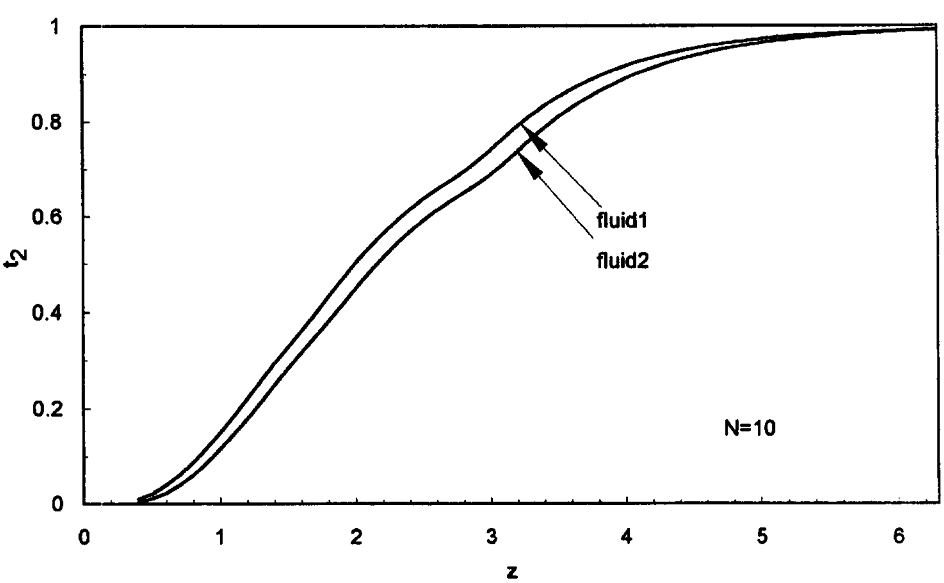

Fig. 5. Exit temperature response of both the fluids with a step increase in both the fluid inlet temperatures for a U-type plate exchanger. $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(z) = f_2(z) = 1.0$, $N = 10$, $NTU_1 = 1.0, \gamma_w = 0.1$ and $Pe = 5.0$.

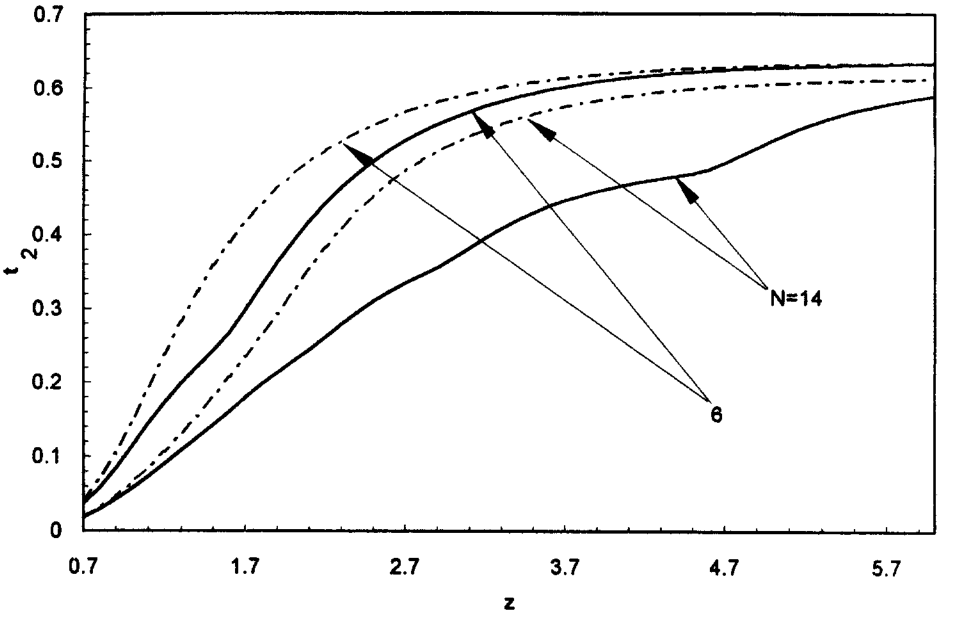

Fig. 6. Comparison of temperature response between the varying velocity and constant velocity (in the port) model in fluid 2. $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0, f_1(z) = 0, f_2(z) = 1.0$, $NTU_1 = 1.0$, $\gamma_w = 0.1$ and $Pe = 5.0$. (———) Varying velocity model, (–·–·–·) constant velocity model.

calculated for both the present model of 'phase lag' [equations (13)–(21)] and a simple 'phase lag model', where the velocity of fluid is assumed to be constant, not only in the channels, but also in the port between them. This physically means that the port which carries the fluid to the channels is a conical one, so that the loss of velocity due to departure of fluid at the channels is compensated by the gain in velocity due to its decreasing cross-section. Under such an assumption the phase lag can be expressed by the simple expression

$$\phi_i = \frac{l_i/V_g}{\tau_{r1}}$$

where V_g is the constant velocity in the port. It has been observed in Fig. 6 that the temperature swing of the hot fluid disappears under such an assumption, even for U-type configuration. From this it can be inferred that this swing in temperature response is due to the decreasing velocity of the fluid in the original model, which gives rise to more 'phase lag', and it becomes prominent when such phase lags get added in a U-type heat exchanger with the fluid which undergoes a temperature rise at the inlet. With constant fluid velocity in the port, the phase lag remains at a lower order of magnitude and the swing in temperature cannot be observed.

The effect of dispersion on the cold and hot fluid outlet temperatures has been enumerated with Fig. 7. Here, for a plate exchanger with six channels, the transient responses for Péclet number in the range $2.0 < Pe < 20.0$ has been calculated. The hot side temperature is found to decrease with the increase in Péclet number, while that of the cold side is observed to increase with it. This can be well explained by the fact that dispersion is virtual conduction in fluid, and hence it degrades the thermal performance of the equipment.

The temperature responses due to simple sinusoidal oscillation in the inlet fluid temperature of side 2 have been presented in Fig. 8. It is interesting to note that the uneven temperature rise which was obtained in step response is not present here because of the fact that, whatever the phase lag, the response from each channel will be a sinusoid and they add up to give another sinusoid. The phase lag here only contributes to the phase shift and the amplitude attenuation of this sinusoid. Figure 8(a) and (b) shows that this is manifested as the difference between U- and Z-type exchangers, which increases with increasing number of channels. Here also the initial response lag is found to be less for U-type exchanger than Z-type, which is quite logical. The effect of Péclet number on sinusoidal response is depicted in Fig. 9. It is found that the increase in Péclet number attenuates the amplitude of oscillation in the hot side, while it increases the amplitude of the cold side.

The effect of other parameters, such as NTU and R_2 is not presented here due to space limitation. They also show the expected behaviour with dispersion playing an increasingly greater role with increasing values of NTU.

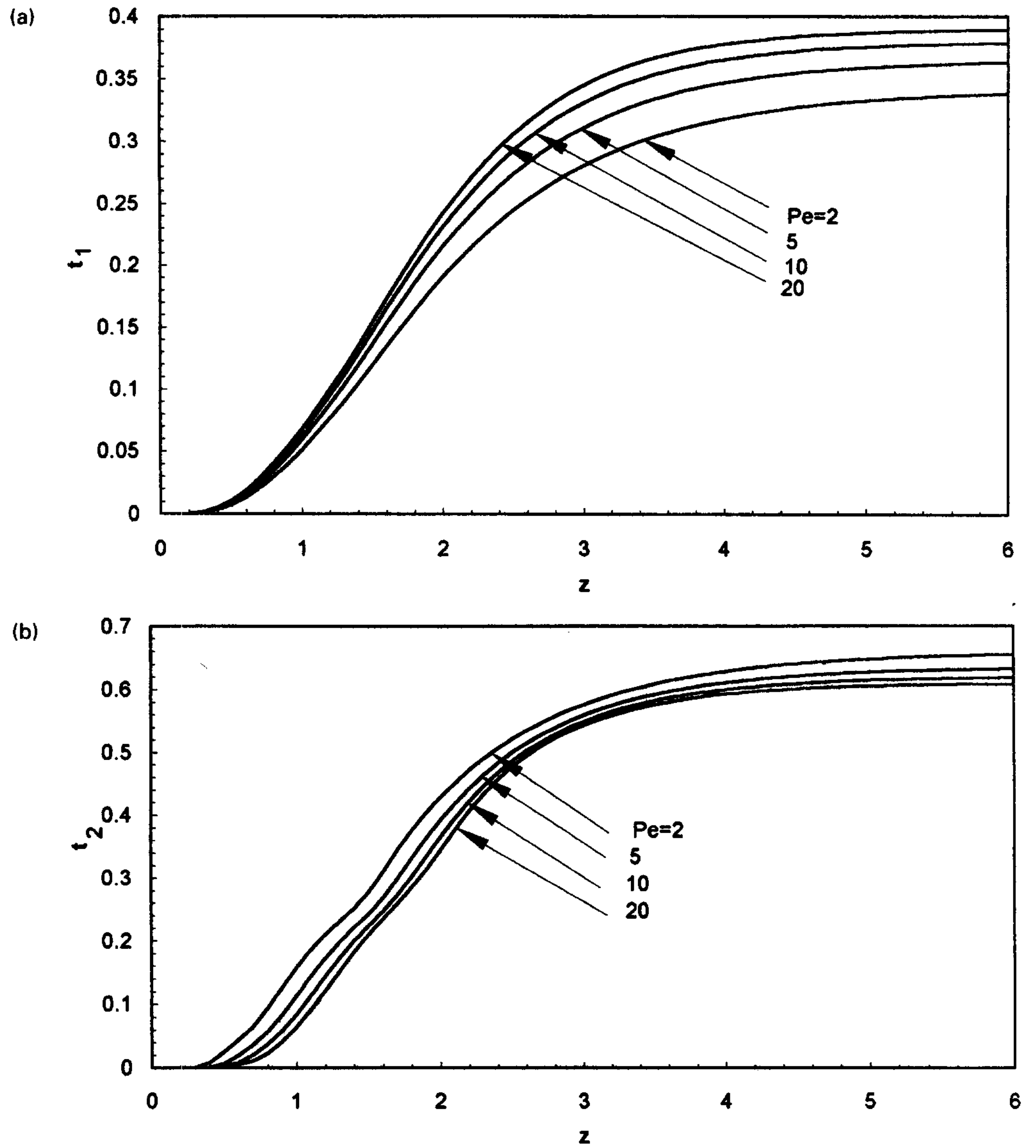

Fig. 7. Effect of Péclet number on the exit response due to a step change of inlet temperature of side 2 in a U-type exchanger. $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $R_N = 1.0$, $f_1(z) = 0.0$, $f_2(z) = 1.0$, $NTU_1 = 1.0$, $\gamma_w = 0.1$ and $N = 6$. (a) Fluid 1, (b) fluid 2.

CONCLUSIONS

In order to predict temperature transients in a plate type heat exchanger an analysis has been presented with dispersion in the fluids so as to take care of the flow maldistribution. The method is efficient and versatile in predicting the transient behaviour which may occur due to step, sinusoidal or any other disturbance. It is important that the model takes care of the longitudinal conduction in solid plates and the heat capacities of the solid and fluids. With a dispersive Péclet number introduced to take care of the deviation from plug flow in the channels, the model formulates the problem in the form of a set of partial differential equations which are solved by using Laplace transform. To obtain the responses in the real time domain, numerical inversion of the Laplace transform has been utilized.

The plate heat exchangers are different from shell-and-tube heat exchangers in the sense that fluids enter the channels of such exchangers with increasing phase lag. The effect gets multiplied in a U-type exchanger. The present analysis incorporates this effect and it is observed that the effect increases with the number of

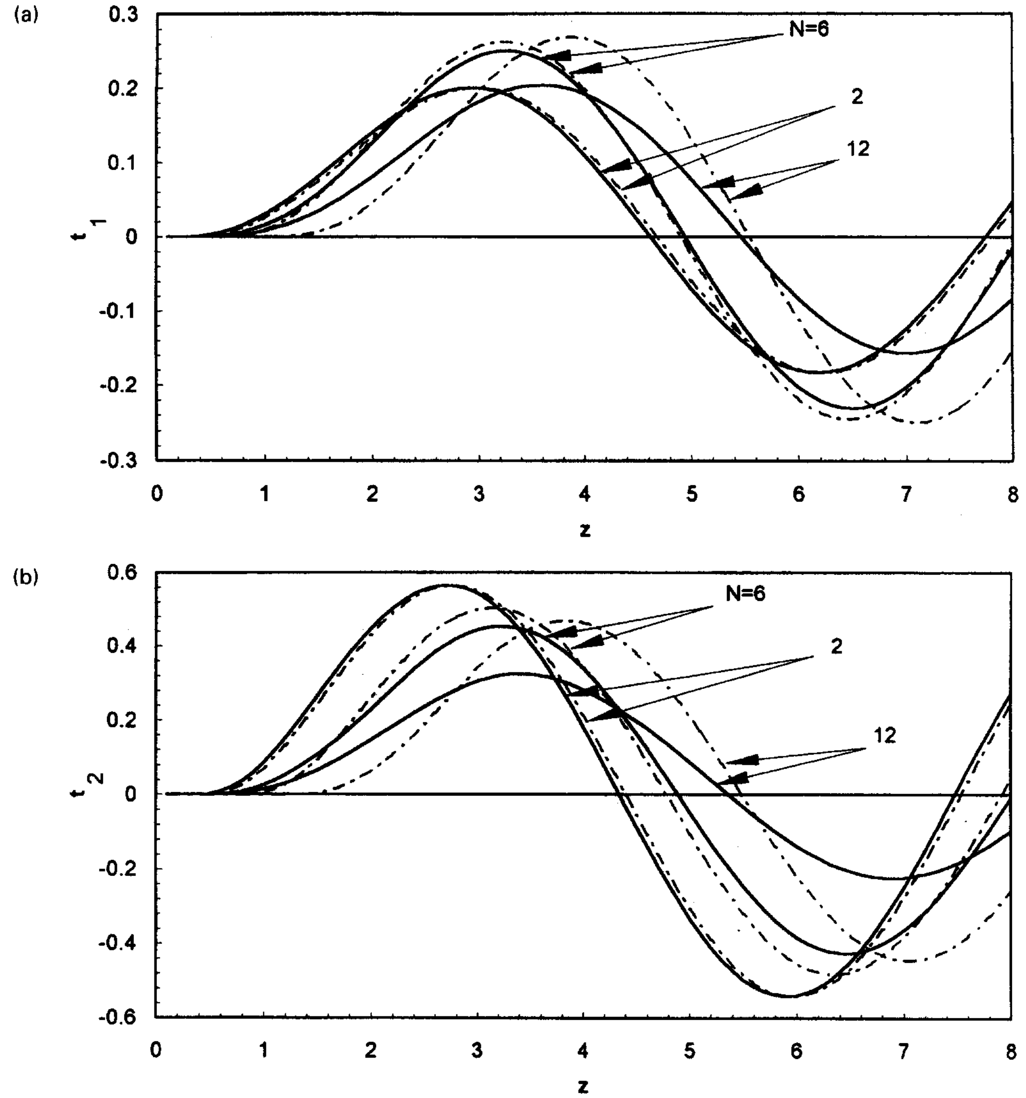

Fig. 8. Effect of number of channels on the exit response due to a sinusoidal variation of the inlet temperature of side 2. $R_w = 0.2$, $R_\tau = 1.0$, $f_1(z) = 0.0$, $f_2(z) = \sin z$, $NTU_1 = 1.0$, $\gamma_w = 0.1$ and $Pe = 5.0$. (a) Fluid 1, (b) fluid 2; (———) U-type, (-·-·-·) Z-type.

channels for both step and sinusoidal response. The manifestation of the effect is observed in the form of difference in response between U- and Z-type exchangers. It is important to note that, when the decreasing velocity in the gasket port is accurately calculated, a swing in temperature rise is observed in the fluid in which step change takes place at the entry. The influence of flow maldistribution is found to result in degradation of thermal performance of the heat exchanger. Péclet number is the quantitative indicator of this effect, which affects the temperature history resulting in either amplitude attenuation or decreasing steady-state temperature, depending on the type of temperature transient that has taken place. The analysis clearly indicates the necessity of incorporating the phase lag and dispersion effects in the transient behaviour of plate heat exchangers irrespective of the type of temperature transient taking place.

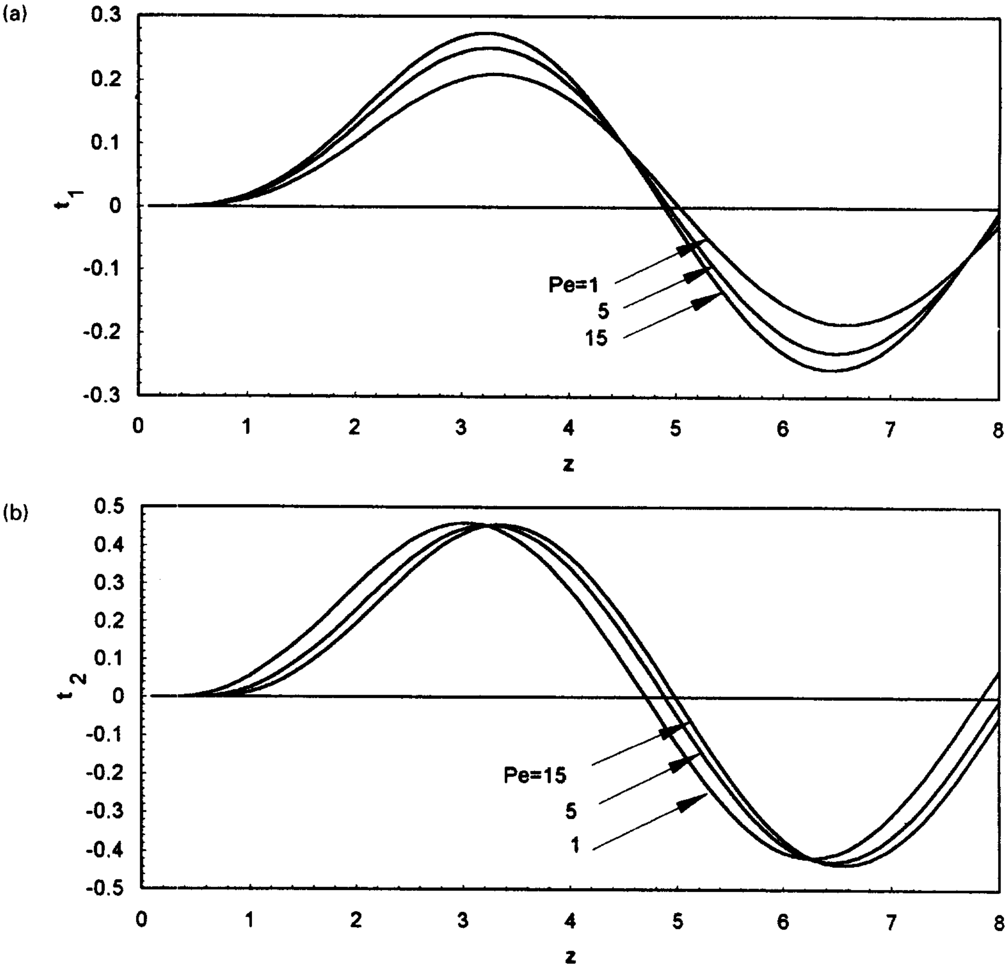

Fig. 9. Effect of Péclet number on the exit response due a sinusoidal variation of the inlet temperature of side 2 in a U-type exchanger. $R_w = 0.2$, $R_\tau = 1.0$, $R_{g2} = 1.0$, $f_1(z) = 0.0$, $f_2(z) = \sin z$, $NTU_1 = 1.0$, $\gamma_w = 0.1$ and $N = 6$. (a) Fluid 1, (b) fluid 2.

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